Analytic Performance Bounds on SAR-Image Target Recognition using Physics-Based Signatures

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Overview

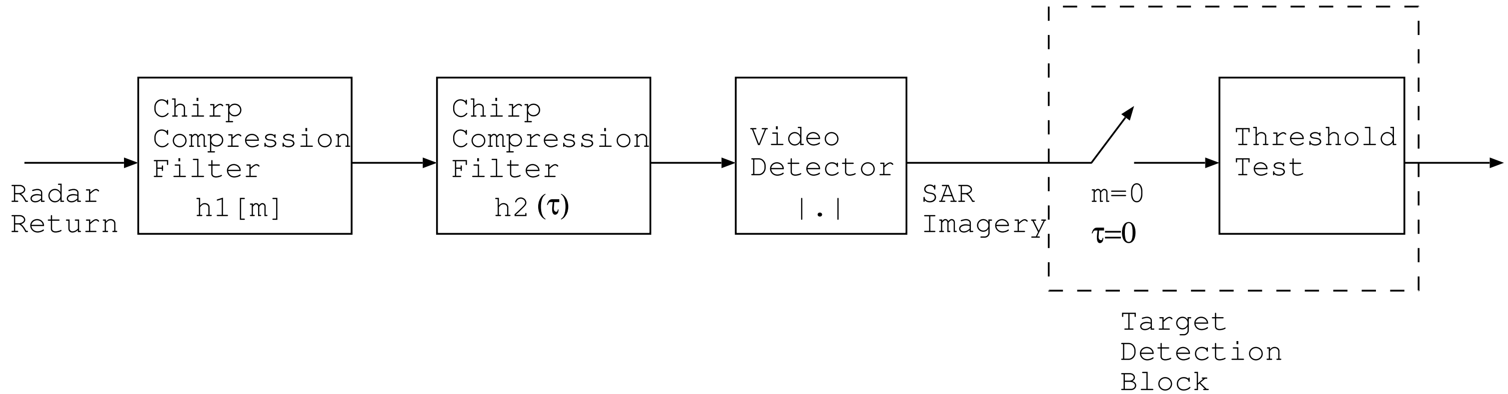
- Target classification performance of SAR-based ATR is assessed based on signal models developed from electromagnetic scattering theory.
- New lower and upper bounds on the probability of correct classification (PCC) are developed.
- Performance discrepancy of conventional full-resolution processor with respect to an optimum whitening-filter processor is discussed.

Radar Signal Model

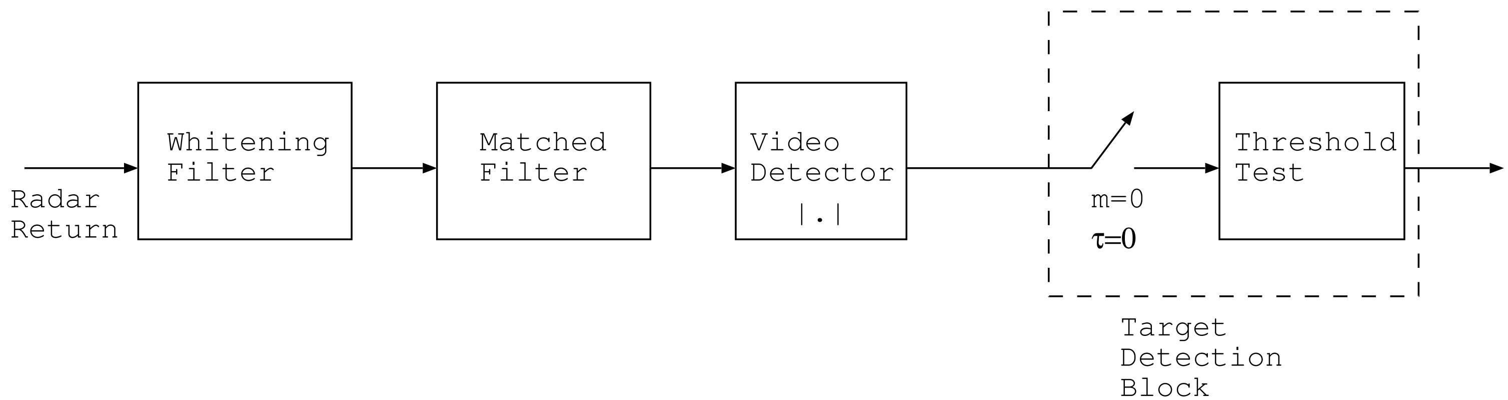
- Radar return signal models are developed via an electromagnetic scattering theory
- Target return
  - Specular mirrors: perfectly conducting square flat plates
  - Dihedrals: pairs of perfectly-conducting plates that meet at right angles
- Clutter
  - Radar returns from non-target reflection
  - Modeled as reflection from an extended rough ground surface
- Receiver noise
  - Zero-mean white Gaussian noise

SAR Processor Models

- Conventional processor



- Whitening processor



Target Classification

- Return signal under H_i:

$$\mathbf{r}(m,\tau) = \sum_{p^i=1}^{M_i} e^{i\phi_{p^i}} \mathbf{s}_{p^i}(m - m_{p^i}, \tau - \tau_{p^i}) + \mathbf{w}(m,\tau)$$

- Orthogonality condition prevails:

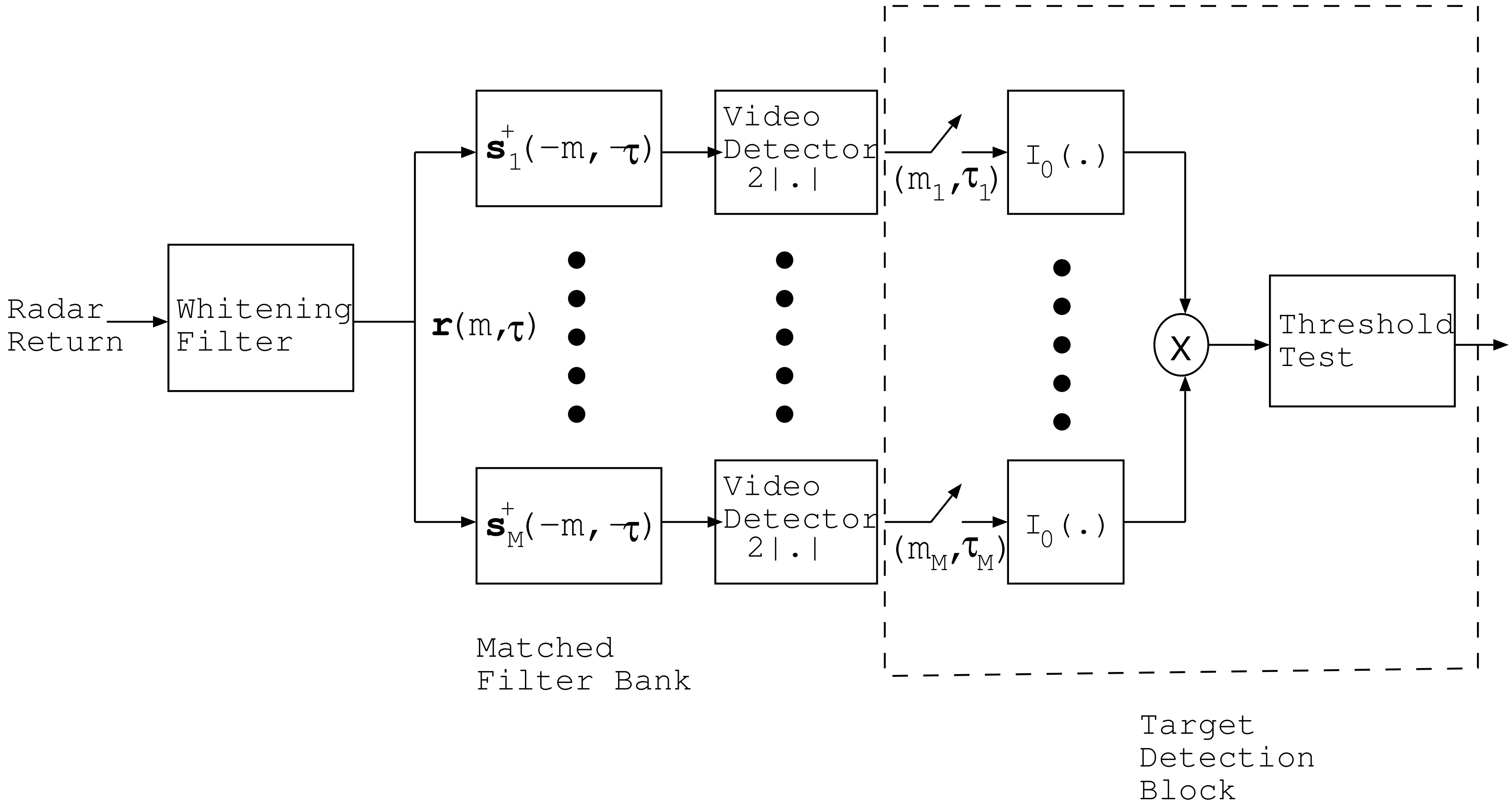
$$\sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} d\tau \ \mathbf{s}_i^{\dagger}(m - m_i, \tau - \tau_i) \cdot \mathbf{s}_j(m - m_j, \tau - \tau_j) \approx 0$$

- Maximum aposteriori probability (MAP) rule involves:

$$l_i(\mathbf{r}) = \frac{p_{\mathbf{r}|H_i}(r_1, r_2, ..., r_{M_i}|H_i)}{p_{\mathbf{r}|H_0}(r_1, r_2, ..., r_{M_i}|H_0)} = \prod_{m=1}^{M_i} \exp[-E_m] I_0(2|r_m|)$$

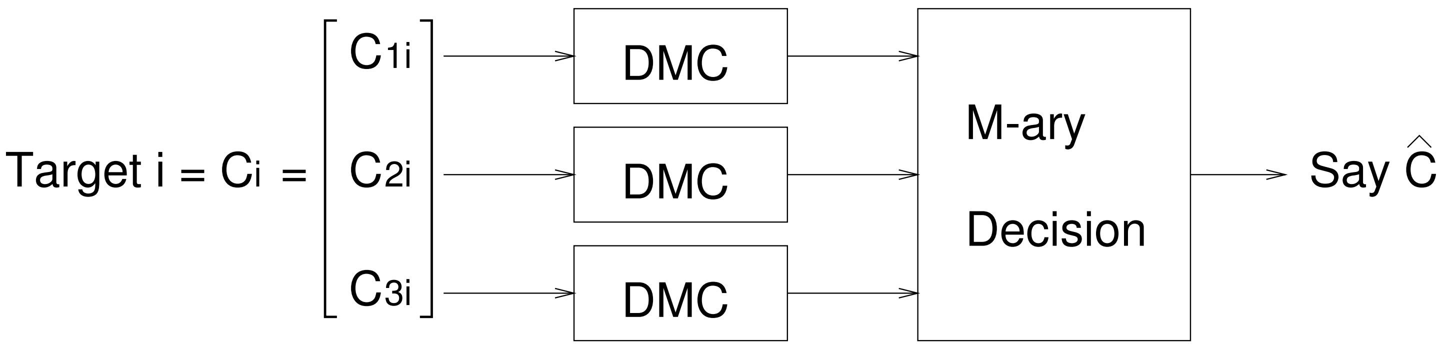
Target with Known Reflector Positions

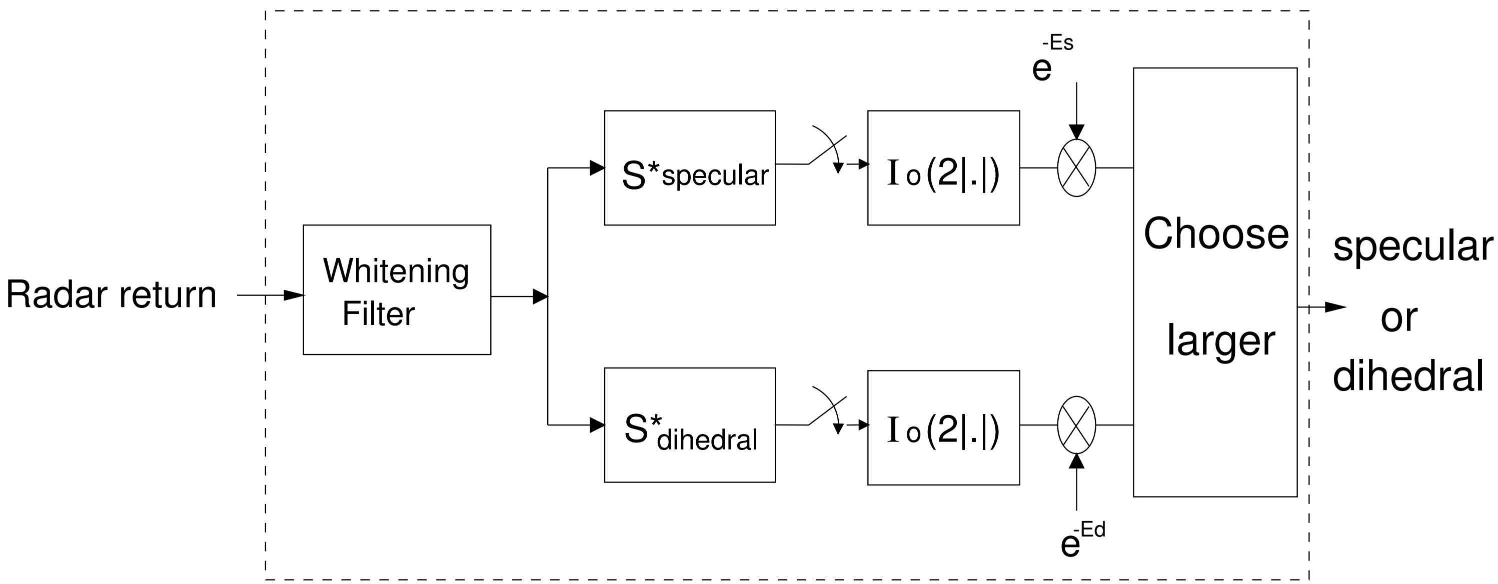
- Classification scheme



Lower bound on PCC

- Find PCC for sub-optimal component-wise classifier





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Upper bound on PCC

- Assume exact phases are given

$$\Pr(\mathit{error} \mid H_i) \geq \Pr(\bigcup_{j \neq i} \mathcal{E}_{ij} \mid H_i,\ \mathit{phase\ given}),$$

where $\mathcal{E}_{ij} = \{\|\mathbf{r}' - \mathbf{s}_j\| \leq \|\mathbf{r}' - \mathbf{s}_i\|\}$

- De Caen's inequality lower bound:

$$\Pr(\mathit{error} \mid H_i,\ \mathit{phase\ given})\ \geq\ \sum_{j \neq i} \frac{Q^2(d_{ij}/2)}{\sum_{k \neq i} \Psi(\rho_{jk}, d_{ij}/2, d_{ik}/2)}$$

where

$$d_{ij} = \|\mathbf{s}_i - \mathbf{s}_j\|,\ \ \rho_{jk} = \frac{< \mathbf{s}_i - \mathbf{s}_j, \mathbf{s}_i - \mathbf{s}_k >}{\|\mathbf{s}_i - \mathbf{s}_j\| \|\mathbf{s}_i - \mathbf{s}_k\|},$$

$$\Psi(\rho, a, b)\ =\ \frac{1}{2\pi\sqrt{1-\rho^2}} \int_a^\infty \int_b^\infty \exp(-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}) dx\, dy.$$

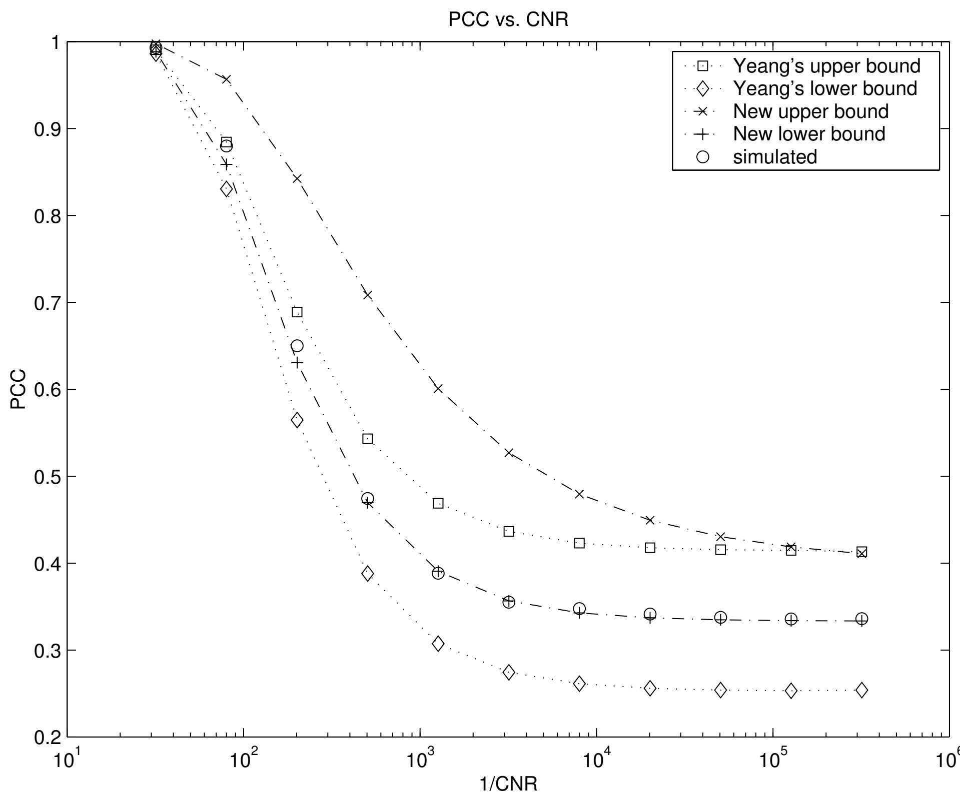
Radar Parameters

Flight Parameters	Radar Parameters	Reflector Parameters
aircraft altitude $L = 5000\,\mathrm{m}$	antenna radii $a_x = a_y = 1\,\mathrm{m}$	relative permittivity $\epsilon_r = 10 + i5$
aircraft speed $v = 100\,\mathrm{m/s}$	Tx and LO powers $P_T = P_{LO} = 1\,\mathrm{W}$	HV clutter strength $\epsilon = 0.2$
slant angle $\psi = 45^\circ$	radar frequency $f_c = \Omega_c/2\pi = 10\,\mathrm{GHz}$	HH×VV correlation $\rho = 0.57$
	pulse-repetition period $T_s = 10\,\mathrm{ms}$	
	pulse width $T_0 = 0.05\,\mu\mathrm{s}$	
	chirp bandwidth $W_0 = 100\,\mathrm{MHz}$	

Example

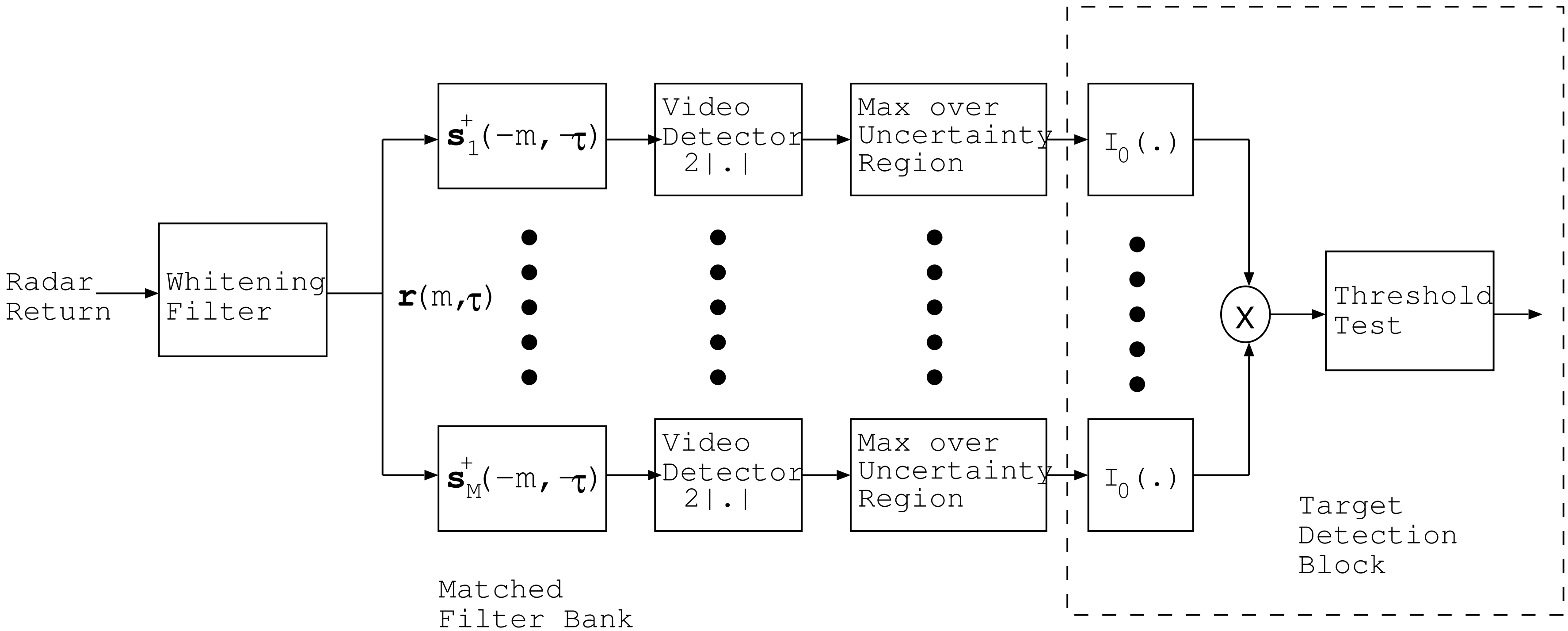
- Previous PCC bounds (Yeang & Shapiro 1999) rely on special symmetries.
- New PCC bounds make no symmetry assumptions.

	Target 1	Target 2	Target 3
1	S(0, 0)	S(0, 0)	S(0, 0)
2	S(-7, -3)	S(-7, -3)	D(-7, -3)
3	S(5, -5)	D(5, -5)	D(5, -5)



Target with Uncertain Reflector Positions

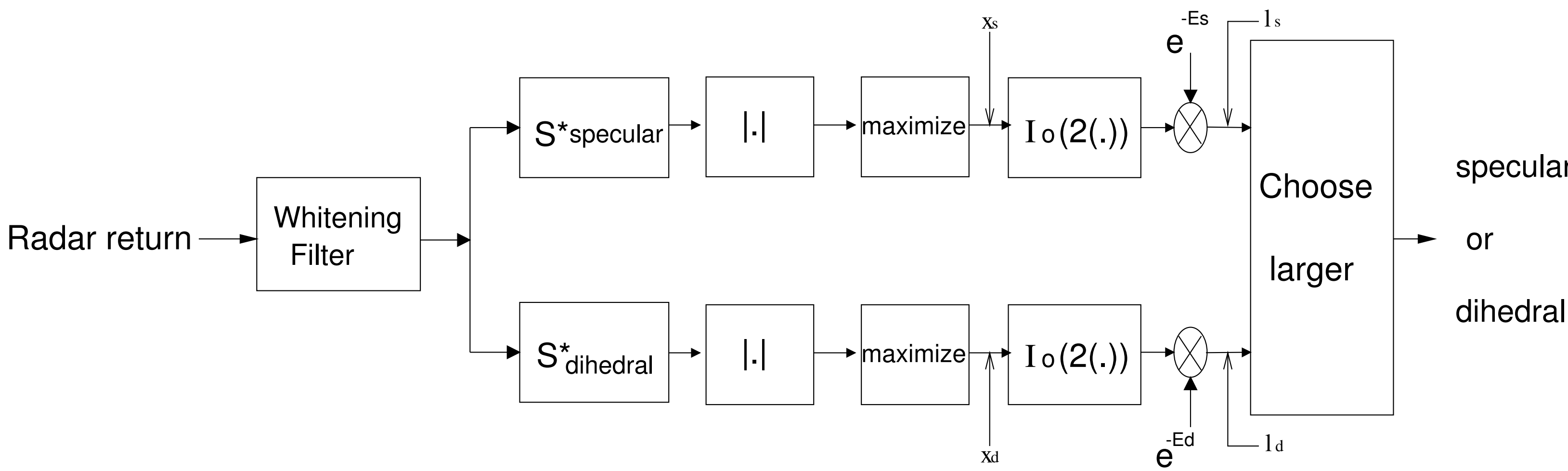
- Classification scheme



- Performance analysis leads to level-crossing theory.

Target with Uncertain Reflector Positions

- Lower bound on PCC: use component-wise classifier



$$\Pr(say\ spec|spec) = 1 - \int_0^\infty dX_d\, p_{x_d|spec}(X_d \mid spec) \Pr(x_s < \gamma \mid spec)$$

where $\gamma = \frac{1}{2} I_0^{-1}(e^{E_s - E_d} I_0(2X_d))$.

$$\Pr(x_s < \gamma \mid spec) = \mathrm{P_1}^{N-1}(\gamma)\mathrm{P}_2(\gamma)$$

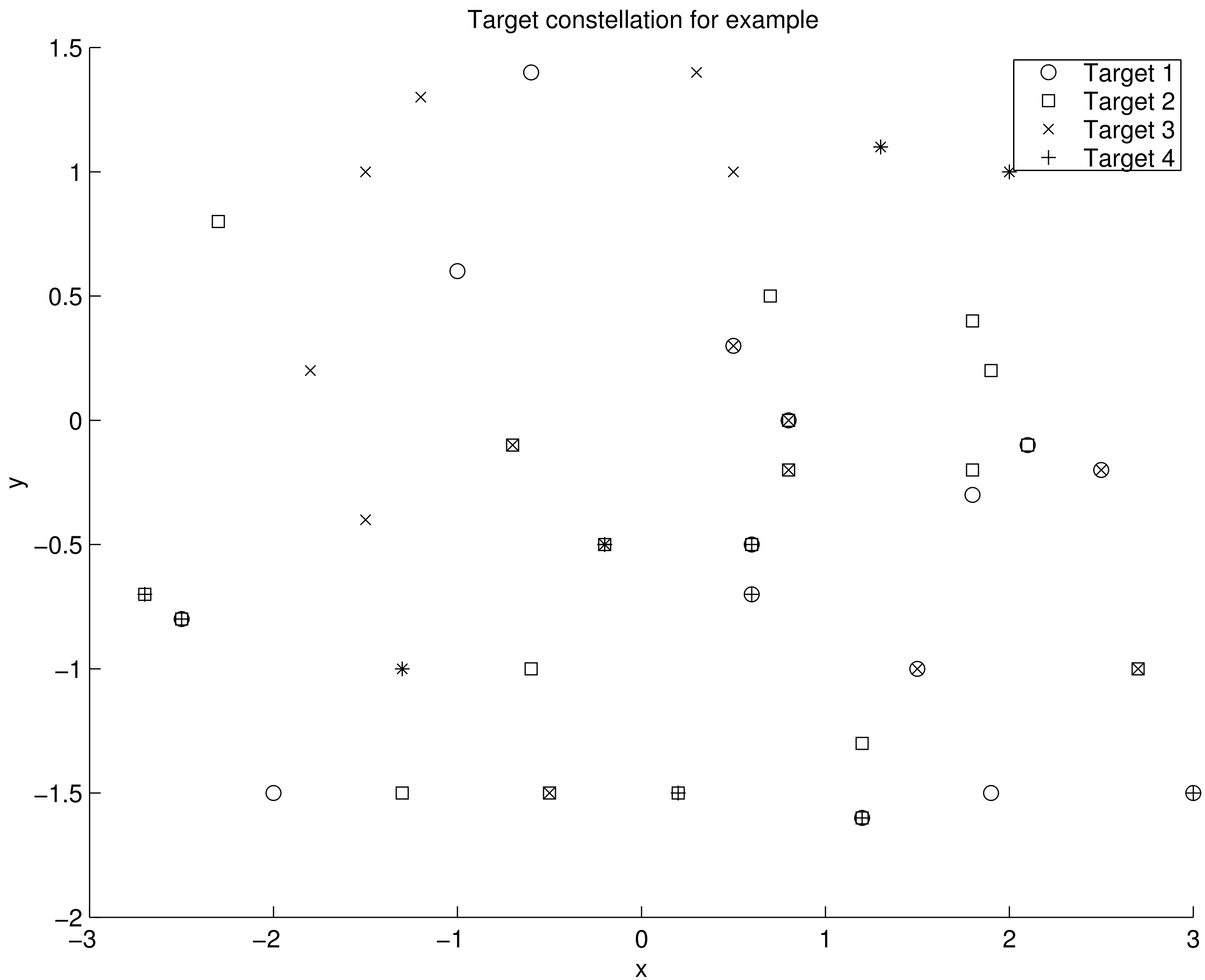
Target with Uncertain Reflector Positions

- Upper bound on PCC

$$\mathrm{GLLR}_k = \sum_{p^k=1}^{M_k} \left(-E_{p^k} + 2 \max \Re\{ \sum_m \int d\tau \mathbf{s}^*_{p^k}(m - m_{p^k}, \tau - \tau_{p^k}) \cdot \mathbf{r}(m, \tau) \} \right)$$

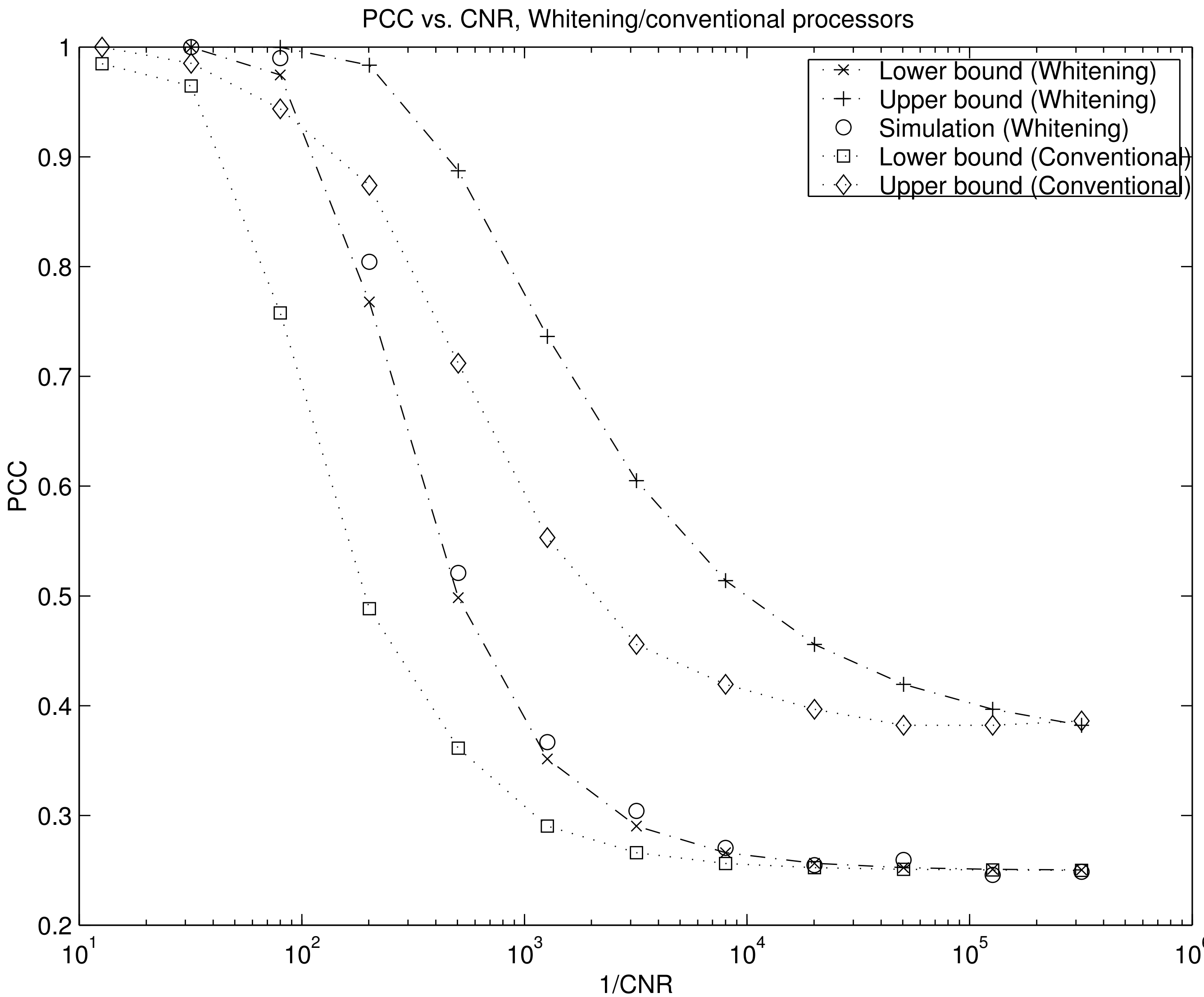
$$\begin{aligned} \mathrm{PCC}|H_i &\le 1 - \Pr\Big(\bigcup_{\forall j \neq i} \mathrm{GLLR}_i < \mathrm{GLLR}_j \mid H_i \textit{ phase given} \Big) \\ &\le 1 - \max_{j \neq i} \Pr(\mathrm{GLLR}_i < \mathrm{GLLR}_j \mid H_i \textit{ phase given}). \end{aligned}$$

Example

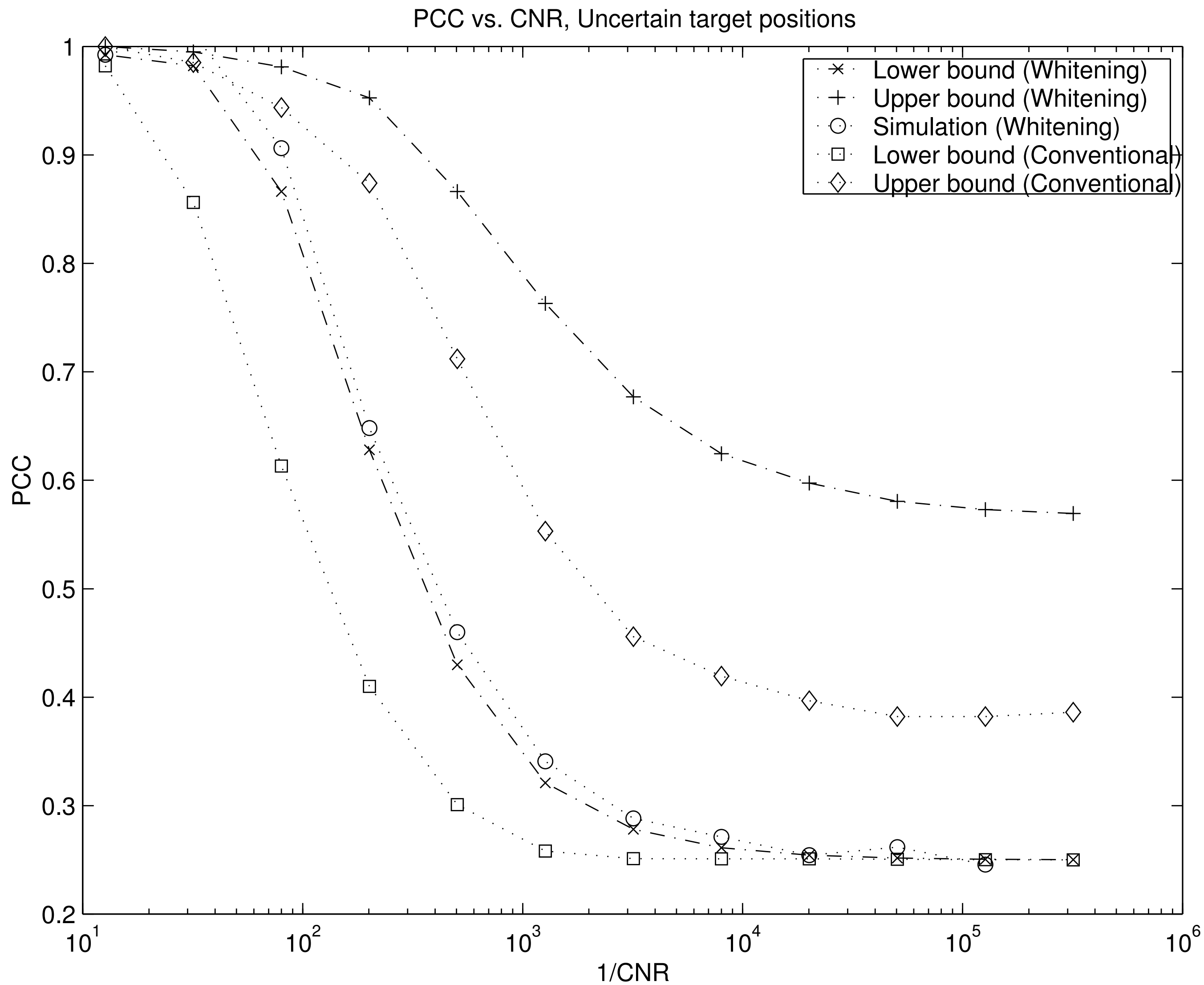


Target constellation for example
Target 1
Target 2
Target 3
Target 4
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−3
−2
−1
0
1
2
3
x
y

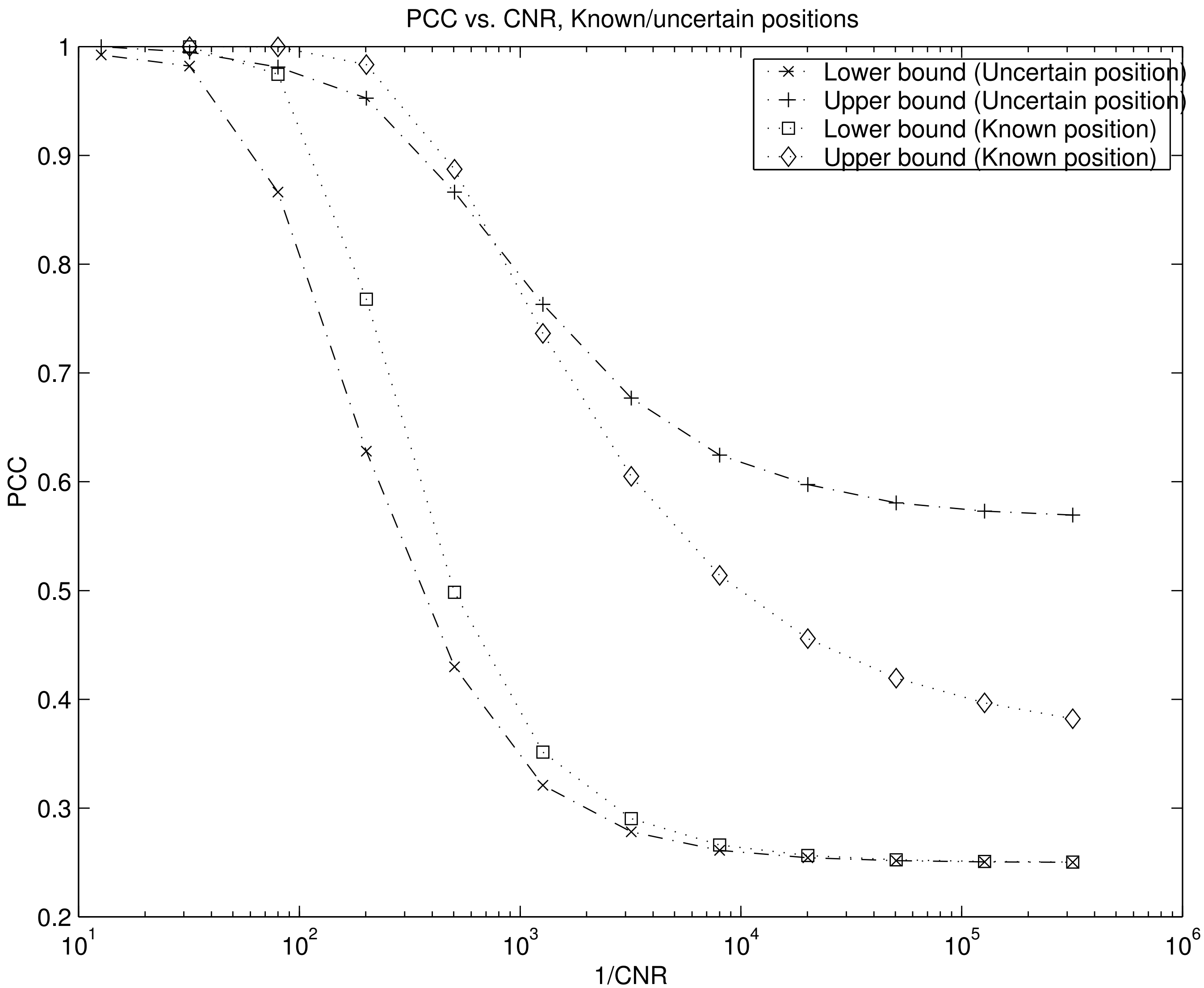
Known-Position Performance



Uncertain-Position Performance



Uncertain-Position Performance



Conclusion

- Lower bound on PCC developed by component-wise decisions
- Upper bound obtained by assumption that relative phases are known
- Whitening-filter processor has about 6 dB gain in SNCR to the conventional full-resolution processor